Two Anchors for the Attractor Framework: Hydrogen and the Jeans Instability Application Paper – June 2026 [A] (Application)

Abstract

The attractor framework has been extended beyond the original variables of basin depth (B) and corrective permeability (κ) to include **energy barrier (B_E)** , **threshold depth (B_T)** , and **channel accessibility (C)** . This paper provides empirical anchoring for these extensions using two well-understood physical systems: the hydrogen atom and the Jeans instability of a gas cloud. Hydrogen's 2p and 2s transitions have identical B_E (10.2 eV) yet differ in κ by eight orders of magnitude. This demonstrates that B_E alone is insufficient; a second parameter (C) is required. The ratio of their Einstein A-coefficients is independently predicted by quantum electrodynamics (dipole vs. two-photon processes), providing a non-circular check of the factorised form. The Jeans instability provides a contrasting case: a deterministic bifurcation where the collapse threshold is a **threshold depth B_T = M/M_J − 1** (for M > M_J). The linear growth rate of the instability scales as Γ∝BTΓ∝*BT*□□, a power law, in contrast to the exponential Arrhenius form of hydrogen. Together, these two test cases validate the extended attractor framework across both noise-driven escape and deterministic bifurcation regimes, using a shared vocabulary (B_E, B_T, C, κ) while acknowledging that each regime draws on the appropriate subset.

1. Introduction

The attractor framework originally described persistence using basin depth B and corrective permeability $\kappa = 1/\tau$. However, the hydrogen atom revealed a critical limitation: two states with identical B (the 2p and 2s levels) have vastly different κ. This forced the introduction of **channel accessibility (C)** , leading to the extended expression for noise-driven escape: $\kappa_{i\to j} = \nu_0 \, C_{ij} \, e^{-B_{E,ij}/\sigma}$

where B_E is the energy barrier, σ is noise (e.g., kT), and ν_0 an attempt frequency. For deterministic bifurcations (e.g., gravitational collapse of a gas cloud), a different descriptor is needed: **threshold depth (B_T)** , with κ (or the growth rate of the instability) following a power law rather than an exponential. This paper demonstrates that both extensions are empirically grounded, using hydrogen to illustrate the need for C and the Jeans instability to illustrate the need for B_T.

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2. Hydrogen: The Need for Channel Accessibility C

2.1 Data

Transition	**B_E (eV)**	**κ (s^{-1})**	**Measured A-coefficient**	**Process**
2p → 1s	10.2	6.26×10^{8}	6.26×10^{8} s^{-1}	Electric dipole (E1)
2s → 1s	10.2	8.22	8.22 s^{-1}	Two-photon (E1E1)

2.2 Why B_E Alone Fails

Both states have the same energy barrier to the ground state (10.2 eV), yet their decay rates differ by eight orders of magnitude. This shows that the basin depth B (here represented by B_E) is insufficient to determine κ; a second parameter must be introduced.

The framework defines **C** as a dimensionless channel accessibility. For a given transition mechanism (e.g., electric-dipole), C is the ratio of the actual transition probability to the theoretical maximum for that mechanism. For the 2p → 1s E1 transition, we set C = 1. The 2s → 1s decay is not an E1 transition at all; it proceeds via a different physical process (two-photon emission). Its rate is independently calculated from quantum electrodynamics without reference to the framework. The ratio of the two measured rates ($\approx 10^8$) is predicted by QED and is not a free parameter. Therefore, the factorised form κ ∝ C e^{-B_E/σ} with B_E identical implies that C must account for the entire rate difference. This is consistent with the independent QED prediction, providing a non-circular validation that an additional channel-dependent parameter is needed.

Note: The 2s→1s process is not a suppressed version of the same channel; it is a different channel (two-photon vs. single-photon). For the purpose of validating the need for a channel-specific parameter, this is sufficient. The framework's C parameter is better illustrated by comparing allowed E1 transitions with different matrix elements (e.g., 2p→1s and 3p→1s), where the same mechanism applies and the ratio of C values is independently known. In any case, hydrogen irrefutably demonstrates that B_E alone does not determine κ.

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3. Gas Cloud (Jeans Instability): Threshold Depth and Power-Law Scaling

3.1 The Bifurcation Regime

A uniform, isothermal, self-gravitating gas cloud of mass M has a critical **Jeans mass** M_J. For M > M_J, the cloud is unstable to gravitational collapse; for M < M_J, it is stable. The transition is a **saddle-node bifurcation** in the dynamical landscape.

3.2 Attractor Variables for a Deterministic Bifurcation

- **Threshold depth**: $B_T = M/M_J - 1$ (for M > M_J). At $B_T = 0$ the bifurcation occurs.
- **Energy barrier**: For a deterministic bifurcation, there is no thermal barrier; B_E is not defined. The transition is controlled solely by the distance to threshold.
- **Growth rate**: For M > M_J, the linear growth rate Γ of the instability is the inverse of the collapse time. This serves as the analogue of κ in this regime.

3.3 Scaling Law from Linear Stability Analysis

The standard Jeans dispersion relation for a self-gravitating, isothermal medium gives: $\omega^2 = k^2 c_s^2 - 4\pi G\rho_0$,

where $c_s = \sqrt{kT/(\mu m_H)}$ is the sound speed and ρ_0 the background density. For a cloud of mass M, the critical wavenumber is $k_J = \sqrt{4\pi G\rho_0}/c_s$. For M > M_J, the longest wavelength (smallest k) is unstable, and the growth rate is $\Gamma = \sqrt{4\pi G\rho_0 - k^2 c_s^2}$.

Near the threshold, the deviation can be expressed in terms of B_T. Using the relation between cloud size and density, one finds $\Gamma \propto \sqrt{B_T}$. Hence the collapse time $\tau \sim 1/\Gamma \sim B_T^{-1/2}$. This is a power law with exponent 1/2, in contrast to the exponential Arrhenius form of hydrogen.

On the stable side (M < M_J), the frequency ω is real, giving oscillatory sound waves. Without a dissipative mechanism, there is no exponential recovery; thus the concept of a "recovery rate" κ is not directly applicable. The framework's threshold depth B_T is best understood as a control parameter on the unstable side.

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4. Synthesis: Shared Vocabulary, Distinct Descriptors

Feature	**Hydrogen**	**Jeans Instability**
Regime	Noise-driven quantum escape	Deterministic bifurcation
Primary descriptor	B_E (energy barrier)	B_T (threshold depth)
Second descriptor	C (channel accessibility)	Not required (power-law exponent fixed)
Scaling	Exponential: $\kappa \propto C e^{-B_E/\sigma}$	Power law: $\Gamma \propto B_T^{1/2}$

Both systems are described by the same conceptual **vocabulary** (basin depth, corrective permeability, threshold, accessibility), but each regime draws on the appropriate subset. Hydrogen validates the need for a channel-specific factor C, while the Jeans instability validates the concept of a threshold depth B_T and the associated power-law scaling.

5. Conclusion

The hydrogen atom and the Jeans instability provide empirical support for the extended attractor framework. Hydrogen shows that identical energy barriers can yield vastly different transition rates, necessitating a channel accessibility parameter C. The Jeans instability shows that deterministic bifurcations are governed by a threshold depth B_T and follow power-law scaling, distinct from the exponential Arrhenius law. Together, these two test cases anchor the framework across two fundamental classes of attractor transitions. The next step is to extend the approach to dissipative systems and to social/cognitive attractors, where C may become state-dependent and network-derived.

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Archetypes as Strange Attractors: Conceptual Parallels with the Attractor

Framework

Robert Galida
Independent Researcher
June 2026
fantasyattractor.com

Abstract

The attractor framework proposes that persistence under perturbation is the fundamental mark of reality, with corrective permeability (κ) serving as a proposed measure of a system's capacity to return to its attractor after perturbation. Van Eenwyk (1991) published a paper in the *Journal of Analytical Psychology* proposing that Jungian archetypes function as strange attractors of the psyche—dynamical patterns that organize psychological experience without ever repeating identically. This paper identifies conceptual parallels between Van Eenwyk's archetype-as-attractor model and the attractor framework. Both draw on a shared upstream tradition in chaos theory. Van Eenwyk's model is itself a theoretical analogy, not an empirically validated result; the parallels identified here are therefore conceptual rather than evidential. They demonstrate consistency within a shared intellectual tradition, not independent corroboration. This mapping carries substantially lower evidential weight than the framework's mappings onto quantitatively validated methods such as Symmetric Projection Attractor Reconstruction (SPAR) and the empirically identified hypothalamic line attractor reported by Nair et al. (2023).

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1. Introduction: Archetypes as Dynamical Attractors

The attractor framework (Galida, 2026a, self-published May 2026 at fantasyattractor.com; no DOI) proposes that dissipative attractors—stable configurations toward which systems converge and from which they resist displacement—are the fundamental units of persistent organization across physical, biological, cognitive, and social domains. Corrective permeability (κ) is a proposed measure of a system's capacity to return to its attractor after perturbation.

In 1991, John Van Eenwyk published "Archetypes: The Strange Attractors of the Psyche" in the *Journal of Analytical Psychology*. Drawing on the emerging science of chaos theory—Gleick, Mandelbrot, Lorenz, Feigenbaum—Van Eenwyk proposed that Jungian archetypes are not fixed images or inherited memories, but dynamical attractors: persistent patterns that organize psychological experience without ever producing identical outputs.

Van Eenwyk's work and the attractor framework were developed entirely independently; neither cites the other. However, both draw on a shared upstream intellectual tradition in chaos theory and nonlinear dynamics. The convergences identified here are therefore expected to some degree: two independent applications of the same mathematical vocabulary to human psychology will naturally produce similar descriptions. This paper identifies conceptual parallels while explicitly distinguishing their evidentiary weight from the framework's mappings onto quantitatively validated methods such as SPAR (Bonet-Luz et al., 2020) and the Nair et al. (2023) line attractor, where Nair et al. empirically identified an approximate line attractor in hypothalamic neural population recordings that encodes an escalating aggressive state.

2. Van Eenwyk's Archetype-as-Attractor Model

Van Eenwyk's central thesis is that Jungian archetypes function as strange attractors of the psyche. He grounds this claim in the formal properties of chaotic dynamical systems:

2.1 Attractors as Organizing Patterns. Van Eenwyk defines an attractor as "the pattern into which a particular motion will settle." Archetypes, he argues, are strange attractors: they organize psychological experience into recognizable, recurring patterns—the hero's journey, the great mother, the shadow—without ever producing identical manifestations.

2.2 Sensitive Dependence on Initial Conditions (SDIC). Drawing on Lorenz's butterfly effect, Van Eenwyk explains individual variation in psychological development: small initial perturbations are amplified geometrically over time, so no two trajectories within an archetypal attractor are identical.

2.3 Bifurcation as Transformation. Van Eenwyk describes the tension of opposites in Jungian psychology as an oscillator. When the tension between consciousness and the unconscious reaches a critical threshold, the system bifurcates—order collapses into chaos, and from that chaos, new patterns emerge. This is the "dark night of the soul"—the necessary intermediate state between an old attractor collapsing and a new one stabilizing.

2.4 Fractal Self-Similarity Across Scales. Van Eenwyk draws on Mandelbrot's fractal geometry. Archetypes exhibit self-similarity across scales: similar themes appear in individual dreams, family dynamics, cultural myths, and religious symbolism. The mandala is a visual representation of a dynamical pattern that recapitulates itself at every level of magnification. It should be noted that “fractal self-similarity” in this context refers to qualitative thematic recurrence across scales, not to the quantitative, measurable property defined in Mandelbrot’s fractal geometry.

2.5 Healthy Chaos vs. Pathological Order. Citing physiological research on heart rate variability, Van Eenwyk argues that healthy systems exhibit chaotic flexibility, not rigid homeostasis. A healthy heart has chaotic variability between beats; a rigid, perfectly regular heart rhythm is pathological. Similarly, a healthy psyche exhibits flexible attractors that can shift in response to perturbation. Loss of variability signals pathology.

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3. Conceptual Parallels with the Attractor Framework

3.1 Archetypes as Attractors. Van Eenwyk’s “strange attractors of the psyche” are descriptively parallel to the attractor framework’s concept of an attractor: a persistent configuration toward which the psyche gravitates and around which it organizes, characterized by self-similarity, resistance to perturbation, and sensitive dependence on initial conditions. The framework generalizes this concept beyond the psyche to physical, biological, and social systems.

3.2 Bifurcation as Basin Transition. Van Eenwyk’s description of bifurcation—the tension of opposites pushing the system to a critical threshold where chaos erupts and new order emerges—is structurally analogous to the framework’s phase transition between attractor basins. The “dark night of the soul” is the chaotic intermediate state between an old attractor destabilizing and a new one forming. The framework describes this same dynamic in climate tipping points, political realignments, and personal cognitive restructuring.

3.3 Healthy Chaos as Corrective Permeability (κ). Van Eenwyk's argument that healthy systems exhibit chaotic variability, not rigid order, is structurally analogous to the framework's corrective permeability (κ). To the extent that κ captures these properties—which has not been formally established—Van Eenwyk's distinction between healthy flexibility and pathological rigidity is consistent with the framework's high-κ/low-κ distinction.

The evidential chain for this parallel should be made explicit. Van Eenwyk's source is physiological research on heart rate variability (HRV)—a finding about cardiac dynamics, not psychological flexibility. Van Eenwyk then extends this to the psyche by analogy. The present paper draws a further analogical connection to κ. The chain is thus three analogical steps removed from its empirical anchor. The parallel is conceptually interesting but rests on layered analogies, not converging evidence.

3.4 Fractal Self-Similarity as Cross-Domain Scaling. Van Eenwyk's use of Mandelbrot's fractal geometry—similar patterns recurring at every scale—is structurally analogous to the framework's claim that attractor dynamics scale across domains. The framework extends this logic beyond the psyche: similar basin dynamics govern biological systems, cardiac electrophysiology, climate systems, political movements, and religious belief. The framework's claim that these dynamics extend to the fundamental structure of physical reality—including the CVU lattice and conservative persistence structures—remains a theoretical assertion under development and is not independently established. In both Van Eenwyk's model and the framework, the cross-domain scaling claim is a qualitative observation of thematic recurrence across scales, not a quantitative demonstration of mathematical fractal structure.

3.5 The Analytic Container as Deliberate Perturbation. Van Eenwyk argues that the therapeutic frame functions to "raise the r value" of the psychological system, pushing it toward the bifurcation point where old attractors destabilize and new ones can emerge. This is structurally analogous to the framework's concept of deliberate perturbation: the analyst, the self-engineer, or the institutional reformer applies targeted perturbations to nudge a system toward a phase transition, knowing that the intermediate chaos is productive, not pathological.

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4. Independence, Shared Lineage, and Evidentiary Weight

Van Eenwyk's work and the attractor framework were developed entirely independently. Van Eenwyk cites Gleick, Mandelbrot, Lorenz, Feigenbaum, and Jung; the framework draws on Ruelle, Prigogine, Olds and Milner, and N=1 self-engineering. Neither cites the other.

However, the shared upstream intellectual lineage in chaos theory substantially limits the evidential weight of these convergences. The vocabulary of chaos theory—attractor, bifurcation, sensitive dependence, fractal—is sufficiently flexible that almost any persistent, complex human phenomenon can be described in these terms. The convergence of two independent applications of this vocabulary may therefore reflect the generality of the vocabulary rather than a discovery about the phenomena themselves. This is a standing methodological limitation that applies to all framework mapping papers using chaos-theory vocabulary, not only to the present paper.

Furthermore, Van Eenwyk's model is itself a theoretical analogy, not an empirically validated result. It was published in a psychoanalytic journal and has not been quantitatively tested. This distinguishes it from the framework's mappings onto the SPAR method (which achieved 96% classification accuracy for a disease-causing genetic mutation) and the Nair et al. line attractor (which was empirically identified in neural population recordings). The present mapping demonstrates conceptual consistency within a shared intellectual tradition; it does not carry the evidential weight of convergence with empirically grounded findings.

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5. Falsifiability Conditions

The following observations would weaken or invalidate the parallels drawn here:

- **Disconfirming observation 1:** If archetypal patterns were shown to be discrete, non-recurring categorical schemas rather than continuous dynamical attractors with sensitive dependence on initial conditions and fractal organization, the attractor model would fail.
- **Disconfirming observation 2:** If the bifurcation model of psychological transformation were shown to be *indistinguishable* from simpler models (e.g., linear stress-response curves, threshold models without chaotic intermediates), the chaos-theoretic interpretation would not be uniquely supported.
- **Disconfirming observation 3:** If quantitative measures of psychological variability—such as linguistic entropy, narrative complexity, or approximate entropy of behavioral time series—showed *no correlation* with therapeutic outcomes or independently assessed psychological health ratings, the healthy-chaos/κ parallel would lose its primary empirical motivation.

Affirmative prediction (long-range): If archetypes function as strange attractors, then therapeutic interventions that successfully transform an individual's relationship to a given archetype should produce measurable shifts in the entropy and complexity of associated psychological content (e.g., dream imagery, narrative patterns, symptom expression). Approximate entropy and sample entropy have been applied to psychological time-series data in existing literature (e.g., Pincus, 1991; Richman & Moorman, 2000) and have been proposed for use in clinical monitoring of mood and behavioral variability. These measures provide a more tractable near-term empirical target than fractal dimension or Lyapunov exponents, which require prior conceptual demonstration that psychological content can be treated as a continuous dynamical time series.

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6. Conclusion

Van Eenwyk's 1991 paper and the attractor framework, developed entirely independently, converge on shared structural descriptions: archetypes are strange attractors—dynamical patterns that organize experience, resist perturbation, exhibit sensitive dependence on initial conditions, and transform through bifurcation. Healthy systems exhibit chaotic flexibility (structurally analogous to high κ); pathological systems exhibit rigid order (structurally analogous to low κ).

These convergences are conceptual, not evidential. Both works draw on the same upstream intellectual tradition in chaos theory, and Van Eenwyk's model is itself a theoretical analogy rather than an empirically validated result. The parallels demonstrate consistency within a shared intellectual tradition, not independent corroboration. The framework remains a self-published, preliminary research program. This mapping is a contribution to its ongoing development, offered with lower evidentiary weight than mappings onto quantitatively validated methods.

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The Persistence Functional: A Mathematical Measure of Attractor Resilience

Author: Robert Galida
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Abstract

The attractor framework says that **persistence under disturbance** is the basic mark of reality.
To turn this idea into a formal science, we introduce the **persistence functional** $P(x)$.

$P(x)$ is a single number that measures:

- How deep a state is inside an attractor basin.
- How quickly it returns after a knock.

We define $P(x)$ for three different kinds of systems:

1. **Deterministic dissipative systems** – here P is linked to Lyapunov exponents and basin stability.
2. **Stochastic systems** – here P is linked to escape time and quasipotential.
3. **Information-theoretic systems** – here P is linked to negative free energy or mutual information.

The **recovery rate** $-\dot{P}/P$ is a universal sign of **critical**

slowing down — a warning that a system is about to tip.

We also discuss limitations: resilience may depend on direction ("anisotropic"), and multiple timescales may need **vector** or **tensor** persistence. We list open mathematical problems.
This paper is a **roadmap**, not a finished theory.

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1. Introduction

In the attractor framework, **persistence under disturbance** is central. But we have not had a single number to say *how persistent* a state is.
The **persistence functional** $P(x)$ aims to fill that gap.

What $P(x)$ should do:

- $P(x)>0$ for states inside an attractor basin.
- For a **conservative attractor** (like a free electron), P is maximal (normalised to 1).
- For a **dissipative attractor**, P drops after a disturbance and then recovers.
  The recovery rate $-\dot{P}/P$ equals:
    - the negative of the largest Lyapunov exponent (for deterministic systems)
    - the inverse return time (for stochastic systems)
    - the rate of information loss (for informational systems)
- P falls as the system approaches a **bifurcation**, giving early warning.

We do **not** give one universal formula. Instead, we give a **family** of definitions, each suited to a different type of system, all united by the same purpose — measuring resilience.

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2. Deterministic Dissipative Systems

Consider a smooth system $\dot{x}=f(x)$ with a stable attractor A and its basin $B(A)$.

A natural candidate for $P(x)$ uses a **Lyapunov function** $V(x)$ — a kind of energy that always decreases inside the basin ($\dot{V}<0$).

We define: $P(x)=1-\frac{V(x)-V_A}{V_{max}-V_A}$

This gives $P=1$ on the attractor and $P\to 0$ at the basin boundary.
Near the attractor, the recovery rate is related to the **largest Lyapunov exponent** λ_1: $-\dot{P}/P\approx-\lambda_1$

When the system approaches a tipping point, $\lambda_1\to 0^-$, so the recovery rate slows down — this is **critical slowing down**.

Conclusion: For deterministic systems, P can be built from a Lyapunov function. The recovery rate equals the negative of the largest Lyapunov exponent.

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3. Stochastic Systems

When noise is present, persistence is about how long it takes to escape from the basin.
The **mean first passage time** $\tau(x)$ — the average time to leave — is a natural measure.

We define: $P(x) = \frac{\tau(x)}{\tau_{max}}$

where τ_{max} is the value at the attractor.

For weak noise, $\tau(x)$ grows exponentially with the **quasipotential** $U(x)$ (Freidlin–Wentzell theory): $\tau(x) \sim e^{U(x)/\epsilon}$

So: $P(x) \propto e^{-(U_{max} - U(x))/\epsilon}$

The recovery rate is the inverse of the return time. As a tipping point is approached, the return time diverges, and the recovery rate goes to zero. This again gives **critical slowing down** – rising variance and autocorrelation.

Conclusion: For stochastic systems, P is proportional to the mean exit time (or the exponential of the quasipotential). This connects persistence to large deviation theory.

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4. Information-Theoretic Systems

For systems where information matters (neural, cognitive, social), we can define persistence using **mutual information** between past and future.

Let $I_{past,future}$ be the **predictive information**. Then: $P(t) = I(\text{past}; \text{future at time } t) \quad \text{or} \quad P = e^{-\text{surprisal}}$

The decay of $P(t)$ over time measures **memory loss**.
Landauer's principle connects information loss to entropy production: $\dot{P}/P \le -\frac{\dot{S}}{k_B \ln 2}$

Alternatively, in the **free energy principle** (Friston), the negative free energy $-F$ acts like a Lyapunov function. We can set: $P = e^{-F/kT} \quad \text{or} \quad P = -F$

Then $-\dot{P}/P$ is the rate of free energy minimisation, which slows near bifurcations.

Conclusion: For information-theoretic systems, P can be defined via mutual information decay or negative free energy, linking persistence to entropy production and predictive coding.

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5. Unifying Recovery Rate and Critical Slowing Down

Across all types of systems, the **recovery rate** $\lambda_{rec} = -\dot{P}/P$ (just after a small disturbance) is a universal indicator:

- **Deterministic dissipative:** $\lambda_{rec} = |\lambda_1|$ (absolute value of the largest Lyapunov exponent)
- **Stochastic:** λ_{rec} = inverse of the return time, related to the quasipotential's curvature
- **Information-theoretic:** λ_{rec} = rate of free energy minimisation or information loss

As the system approaches a bifurcation, $\lambda_{rec} \to 0$. This is **critical slowing down**.
It shows up as rising lag-1 autocorrelation and variance (Scheffer et al., 2009).
So P and its recovery rate give early warnings.

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6. Normalisation for Conservative

Attractors

For a perfect **conservative attractor** (e.g., an electron in its ground state, no decay), the persistence functional should be constant and maximal: $P_{cons} = 1 \text{ for all times}$

No recovery rate is defined (or it is zero). This anchors the scale.

For **emergent approximate conservative systems** (like atomic clocks), P is very close to 1 and decays extremely slowly.

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7. Limitations – Scalar Collapse and Anisotropic Resilience

A single scalar $P(x)$ may not be enough for systems where resilience is **anisotropic** – that is, recovery speed depends on the direction of the perturbation.
High-dimensional systems can have **multiple timescales** (fast and slow modes). A scalar average can miss important structure.

Future work may need:

- **Vector persistence** – a list of recovery rates along different directions.
- **Tensor persistence** – a metric that captures the full shape of the basin.
- **Persistence manifold** – the geometry of the basin in state space.

We accept this limitation. The scalar P is a useful first approximation for systems with isotropic resilience or for early-warning applications where a single number is enough.

For complex systems, a multidimensional generalisation is an open research problem.

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8. Open Mathematical Problems

1. **Derive $P(x)$ from first principles** for a given class of systems (e.g., from a variational principle).
2. **Prove that $-\dot{P}/P = \langle \lambda_1 \rangle$** for a wide class of dissipative systems.
3. **Extend the definition to systems with multiple attractors and chaotic basins** (where basin stability is fractal).
4. **Establish a rigorous relationship between P and the mutual information decay rate** for non-equilibrium processes.
5. **Formulate a universal persistence functional** that works across all regimes — or prove it's impossible.
6. **Test the predictive power of P** in controlled experiments (e.g., ecological microcosms, neural cultures, social media sentiment).
7. **Develop vector/tensor persistence** for anisotropic resilience.

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9. Conclusion

The persistence functional $P(x)$ gives a mathematical language for attractor resilience.

We have given **operational definitions** for three regimes:

- **Deterministic dissipative** → Lyapunov / basin stability

- **Stochastic** → escape time / quasipotential
- **Information-theoretic** → mutual information / free energy

The **recovery rate** $-\dot{P}/P$ unifies critical slowing down across all these domains.

We have explicitly noted **limitations** (scalar collapse, anisotropy) as open problems.

This paper is a **roadmap**, not a final theory. The framework now has a quantitative step.

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