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Robert Galida
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□□□□□$\kappa=\inf_{x\in B\setminus A}\frac{D_\infty(x)}{\delta(x)}$□

□□□□□□□□□□□ $\dot{x}=-Ax$□□□ A □□□□□□□□□□□□□□□□ $\lambda_{\min}(A)$——□□□□□□□□□□□□□□□□□□□□□□□□□□□ $D_\infty(x)\le\delta(x)/\kappa$□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□Koopman□□□□□□□□□□□□□□□□□□□□ κ □□□□□□□□□□□□□□□□□□ D_∞□ □Hamilton-Jacobi□□□□□□□□□□□□□□□□□□□□□□□ □ $\kappa_T=\inf_x \frac{D_T(x)}{\delta(x)}$□ □□□□□□□□

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1. □□

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□□□□□□□□□——□□□□□□ $D_T(x)$——□□ κ□□□□□□□κ □□□□□□□□□□□□□□□□□□□□□□□□ κ □□□□□□□□——□□□□□□□□□□□□□□□□□□□

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2. □□□□□□

□ X □□□□□□□ □·□□·□ □□□□□□□ $\phi_t(x)$ □□□□ $x\in X$ □□□ $t=0$ □□□□□□□□□□□□ $A\subseteq X$ □□□□□□□□□□□□□□□□□□□□□□ B □ A □□□□□

□□□□□□□□□□□□$\delta(x)=d(x,A)=\inf_{a\in A}\|x-a\|$□

□□ 1□□□□□□□□□ □□□□□□ $T>0$□□□□$D_T(x)=\int_0^T \delta(\phi_t(x))\,dt$

□ $T\to\infty$ □□□□□$D_\infty(x)=\int_0^\infty \delta(\phi_t(x))\,dt$

□□ 1□**D_∞**□ □□□□□□□ □□□□□□ $C<\infty$ □ $\mu>0$ □□□□□□ $x\in B$□$\delta(\phi_t(x))\le Ce^{-\mu t}\delta(x)$

□□□□ $x\in B$□$D_\infty(x)<\infty$□

□□□$D_\infty(x)=\int_0^\infty \delta(\phi_t(x))\,dt\le\int_0^\infty Ce^{-\mu t}\delta(x)\,dt=\frac{C}{\mu}\delta(x)<\infty$□□

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□□	□□
□□□	$D_T(x)\ge 0$
□□□	□ $T_2\ge T_1$□ □□$D_{T_2}(x)\ge D_{T_1}(x)$
□□□	$D_{T+S}(x)=D_T(x)+D_S(\phi_T(x))$
□□□□	$\frac{d}{dT}D_T(x)=\delta(\phi_T(x))$
□□□□	$D_T(x)=\int\delta(y)\,d\mu_T(y)$□□□ μ_T□ □□□□□

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3. □□□□□ (κ) □□□

3.1 □□□□

□□ **2**□□□□□□□□$\kappa=\inf_{x\in B\setminus A}\frac{\delta(x)}{D_\infty(x)}$□

□□□ κ □□□□□□——□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□

□□ **κ** □□□□ □□□□□ $\kappa=0$□□□ $D_\infty(x)$ □□□□□ $\delta(x)/D_\infty(x)$ □□□□□□□□□□□□□□□□□□□□□□□□□□ $\kappa>0$ □□□□□□1□□□□□□□□

□□□□□□□□□ κ □□□

3.2 □□□□□□□□□

□□ **1**□□□□□□□□□□□□□ □□□□□ $\phi_t(\alpha x)=\alpha\phi_t(x)$ □□□ t □□□ $\alpha>0$ □□□□□□□□□□□ $\delta(\alpha x)=\alpha\delta(x)$□□□$\frac{\delta(\alpha x)}{D_\infty(\alpha x)}=\frac{\delta(x)}{D_\infty(x)}$□

□

□□$D_\infty(\alpha x)=\int_0^\infty\delta(\phi_t(\alpha x))\,dt=\int_0^\infty\delta(\alpha\phi_t(x))\,dt=\alpha\int_0^\infty\delta(\phi_t(x))\,dt=\alpha D_\infty(x)$

□□□ □□□□□□□□□□□ $x\neq 0$ □□□□□□□□□□□□□□□□□$\kappa=\inf_{\|x\|=1}\frac{\delta(x)}{D_\infty(x)}$□

3.3 □□□□□□□□□

□□ **2**□□□□□□□□□□□□□ □□□ $x\in B\setminus A$□$D_\infty(x)\le\frac{\delta(x)}{\kappa}$□

□□□□□ $1/\kappa$ □□□□□□□□□□□□□□□□ x □□□□□□□□

□□□ □ κ □□ $\delta(x)/D_\infty(x)$ □□□□□□□□□□ x □ $\delta(x)/D_\infty(x)\ge\kappa$ □□□□□ $D_\infty(x)\le\delta(x)\kappa$ □

□□□□□□3□□□□□□□□□□□ v_1 □□ $D_\infty(v_1)=\delta(v_1)/\kappa$ □□□□□□□□□□□□□□□□□□□

3.4 □□□□□□□□□

□□□□□□ $\dot{x}=-Ax$ □□□ A □□□□□□□□□□□□ $0<\lambda_1\le\lambda_2\le\cdots\le\lambda_n$ □□□□□□□□□□□□□□□ $v_1,v_2,\dots,v_n$ □□

□□ $\phi_t(x)=e^{-At}x$ □□□□□ $A=\{0\}$ □□□□□□□□ $\delta(x)=\|x\|$ □□

□□ 3□□□□□□□□ □□ $\dot{x}=-Ax$ □ A □□□□□□ $\inf_{x\neq 0}\frac{D_\infty(x)}{\|x\|}=\lambda_{\min}(A)$

□□□

□□ A □□□□□ e^{-At} □□□□□□□□□□□ $e^{-\lambda_i t}$ □□□□□□□□ $\|e^{-At}\|=e^{-\lambda_1 t}$ □□□□ $x\neq 0$ □ $D_\infty(x)=\int_0^\infty \|e^{-At}x\|\,dt\le\int_0^\infty \|x\|e^{-\lambda_1 t}\,dt=\frac{\|x\|}{\lambda_1}$ □□

□□□□ $\frac{\|x\|}{D_\infty(x)}\ge\lambda_1$ □

□□□□□□□□□ $x=v_1$ □□□□ λ_1 □ □□□□□□□□□□ $\|e^{-At}v_1\|=\|v_1\|e^{-\lambda_1 t}$

□□ $D_\infty(v_1)=\int_0^\infty \|v_1\|e^{-\lambda_1 t}\,dt=\frac{\|v_1\|}{\lambda_1}$ □□□

□□□□ $\frac{\|v_1\|}{D_\infty(v_1)}=\lambda_1$ □

□□□ $\inf_{x\neq 0}\frac{\|x\|}{D_\infty(x)}=\lambda_1$ □□□

□□□ □□□□□□□□□□□□ κ □□□□□□□——□□□□□□□□□□□□□

3.5 [illegible]

[illegible] 4[illegible] [illegible] f [illegible] C^1[illegible] ϕ_t [illegible] C^1[illegible] D_∞ [illegible] $B(A)$ [illegible]$\nabla D_\infty(x)\cdot f(x) = -\delta(x)$

[illegible] [illegible]$D_\infty(\phi_s(x)) = D_\infty(x) - D_s(x)$

[illegible] $s=0$ [illegible] s [illegible]$\frac{d}{ds}D_\infty(\phi_s(x))\big|_{s=0} = -\delta(x)$

[illegible]$\nabla D_\infty(x)\cdot f(x) = -\delta(x)$□

[illegible] [illegible] $f\cdot\nabla D = -\delta$[illegible]Hamilton-Jacobi[illegible]

3.6 [illegible]

[illegible] $\kappa = \inf_x \frac{\delta(x)}{D_\infty(x)}$[illegible] [illegible]——[illegible]

[illegible]κ [illegible] κ [illegible]Lyapunov[illegible]§6[illegible]

3.7 [illegible]

[illegible] $\dot{x} = Ax$[illegible] A [illegible]

[illegible] [illegible]

3.8 □□□□□□□□

□□ 5□□□□□□□□□□□□ □□□□□□□□□□□□□ $\delta(\phi_t(x)) \le Ce^{-\mu t}\delta(x)$

□□□ $x \in B$ □□□□□□□ $C<\infty$ □ $\mu>0$□□□$\kappa \ge \frac{\mu}{C}$□

□□□ □□□□□□$D_\infty(x)=\int_0^\infty \delta(\phi_t(x))\,dt \le \int_0^\infty Ce^{-\mu t}\delta(x)\,dt=\frac{C}{\mu}\delta(x)$

□□□$\frac{\delta(x)}{D_\infty(x)} \ge \frac{\mu}{C}$□

□□□ x □□□□□$\kappa=\inf_x \frac{\delta(x)}{D_\infty(x)} \ge \frac{\mu}{C}$□□□

□□□ □□□□ κ □□□□□□□□ μ/C □□□□□

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4. □□□□□□□□

4.1 Koopman□□

Koopman□□ K_t □□□□□□□□$(K_t f)(x)=f(\phi_t(x))$

□□□□□□ $\dot{x}=-Ax$□Koopman□□□□ $e^{-\lambda_i t}$□□□□□□□□□□□□1□□□□□ □ $e^{-\lambda_1 t}$□□□□□□□□□

□□□□□□□□□□$\rho=e^{-\lambda_{\min} t}$□□□□$-\frac{1}{t}\log\rho=\lambda_{\min}=\kappa$

□□□□□□3□□□□□□□□□□□□□□Koopman□□□□□□□□□□□□□

4.2 □□□□

□□□□□□□□□□□□□□ $(sI+A)^{-1}$ □ $s=-\lambda_i$□ □□□□□□□□□□□□□□ □ $s=-\lambda_1$□□

□□□□3□□ $\kappa=\lambda_{\min}$□□□□□□□□ $s_i=-\lambda_i$□□□□□ □□$\kappa=\min_i$□□(s_i)□

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5. □□□□□□

□□□□□□□□□□□□□□□□□□□□□□□□□□□□$\kappa_T=\inf_{x\in K}\frac{D_T(x)}{\delta(x)}$□

□□ $K\subset B$ □□□ $K\cap A=\varnothing$□

□□ **2**□□□□□□□□ □□□

1. □ $\phi_t(x)$ □ (t,x) □□□□□□
2. $\delta(x)$ □□□
3. □□□□□□ $\delta(\phi_t(x))\le Ce^{-\mu t}\delta(x)$ □□□ $x\in K$ □□□□□□□ $\mu>0$□

□□□□5□□□□□ κ□□□2□□□ $\kappa\ge\mu/C$□□□$\kappa_T\to\kappa$□ $T\to\infty$

□□□□□$\kappa_T-\kappa=O(e^{-\mu T})$

□□□ □□□ $x\in K$□□□□□ □□□$D_\infty(x)-D_T(x)=\int_T^\infty\delta(\phi_t(x))\,dt\le\frac{Ce^{-\mu T}\delta(x)}{\mu}$□

□□ $\delta(x)$ □□□□ K □□□□□ $M=\sup_{x\in K}\delta(x)<\infty$□ □□□$D_\infty(x)-D_T(x)\le\frac{CMe^{-\mu T}}{\mu}$□

□□□ x □□□□ $T\to\infty$ □□□□□□□ $D_T\to D_\infty$□ □ K □□□□□□

□□□□□ KK □□□ K∩A=∅K∩A=∅□δδ □□□□□□ inf□x∈Kδ(x)>0inf x∈K□δ(x)>0□□ □ DT(x)DT□(x) □□□□□□1-2□□□□ TT □□□□□□□§2□□□□□□□□□□ T0>0T0 >0□D∞(x)≥DT0(x)D∞□(x)≥DT0□□(x)□□ DT0DT0□□ □ KK □□□□□□□□□□□□□□□□□□□□□□□□m=inf□x∈KDT0(x)>0m=x∈Kinf□DT0□□(x)>0

□□□inf□x∈KD∞(x)≥m>0x∈Kinf□D∞□(x)≥m>0

DTDT□ □ KK □□□□□□ D∞D∞□ □□□□□ δ(x)/DT(x)δ(x)/DT□(x) □□□□ □ δ(x)/D∞(x)δ(x)/D∞□(x)□□□□□□□□□□□□

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6. □□□□

□□	□□	□□
Q1: □□□□□	inf□δD∞inf D∞□δ□ □□□□□□Lyapunov□□□	□□
Q2: □□□□□□□□	□□□□□□□□□lim□x→Aδ(x)D∞(x)=κlimx→A□D∞□(x)δ(x) =κ □□□□□	□□
Q3: □□□□□	□□□□□□□□□□ AA □□□□□□□	□□
Q4: □□□□□	□□□□□□□□□□□□□□□	□□
Q5: □□□□	□□□□□□□□□□□□□□□	□□
Q6: □□□□	κκ □□□□□□□□□□□□□□□□□□	□□

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7. □□

□□□□□□□□□ DT(x)DT□(x) □□□□□□□□□ κκ□□□□□□κ=inf□xδ(x)D∞(x)κ=xinf D∞□(x)δ(x)□

□□□□□□□□□□□□□□□□□□□□□□ κ=1/τκ=1/τ □□□□□□□□□□□□□□□□ □ D∞(x)≤δ(x)/κD∞□(x)≤δ(x)/κ□□□□□□ κκ □□□□□□□□□□□□□□□□□□□□ D∞D∞□ □Hamilton-Jacobi□□□□□□□□□□□□□□□□□□□Koopman□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□ κTκT□□

□□□□□ □□□□□□κκ □□□□□□□□□□□□□□□□□□□——□□□□□□□□□□□□□□□□□□

□□□□□ □□□□□□□□□□□□Q1-Q2□□□□□□□□Q3□□□□□□□□Q4□□□□□□□□Q5□□

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